Enrollment No:	Exam Seat No:
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# C.U.SHAH UNIVERSITY

# **Summer Examination-2016**

**Subject Name: Engineering Mathematics - IV** 

**Subject Code :4TE04EMT1** Branch:B.Tech (Auto, Mech, EEE, EE, IC, Civil, EC)

Date: 07/05/2016 Marks: 70 Semester: 4 Time: 2:30 To 5:30

#### **Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

#### Q-1 Attempt the following questions:

Write Fourier sine transform of f(t).

(01)

A vector  $\vec{F}$  is solenoidal if

(01)(01)

(14)

In usual notation  $E = 1 - \nabla$ . True or False The function  $\bar{z}$  is not analytic at any point. True or False?

(01)

The function  $e^x \cos y$  is not harmonic. True or False?

(01)

The region  $|z| \le 1$  represent open unit disk. True or False? f)

- (01)
- Range Kutta method is better than Tayler's method. True or False?
- (01)
- The convergence in the Gauss Seidal method is faster than Gauss Jacobi
- (01)

method. True or False? If  $\phi = 3x^2y - y^3z^2$ , find gradient  $\phi$  at the point (1,-2,1) i)

(02)

State Green's theorem. **j**)

(02)

If  $y = 3x^3 - 2x^2 + 1$  find  $\Delta^3 y$ .

## Attempt any four questions from O-2 to O-8

(02)

(05)

#### Q-2 **Attempt all questions**

- (14)
- Find the Fourier cosine integral of  $f(x) = e^{-kx}$  (x > 0, k > 0). Using that evaluate  $\int_0^\infty \frac{\cos \lambda x}{k^2 + \lambda^2} d\lambda$
- Solve the one dimensional wave equation  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}, -\infty < x < \infty, \ t > 0$ b) (05)0 with the initial conditions u(x,0) = f(x),  $\frac{\partial u(x,0)}{\partial t} = g(x)$  and the

boundary conditions  $u, \frac{\partial u}{\partial x} \to 0$  as  $x \to \pm \infty$ .

Find the Fourier transform of f(x) if  $f(x) = \begin{cases} 0 & 0 < x < a \\ x & a \le x \le b \\ 0 & x > b \end{cases}$ c) (04)





#### Q-3 Attempt all questions

- a) Determine analytic function whose imaginary part is  $e^x (x \cos y y \sin y)$  (05)
- b) If f(z) = u + iv is an analytic function of z and  $u + v = e^x (\cos y + \sin y)$ , find f(z). (05)
- c) Find p such that the function  $f(z) = r^2 \cos 2\theta + i r^2 \sin p\theta$  is analytic. (04)

### Q-4 Attempt all questions

(14)

a) Under the transformation  $W = \frac{1}{z}$ 

(05)

- i. Find the image of |z 2i| = 2.
- ii. Show that the image of the hyperbola  $x^2 y^2 = 1$  is the lemniscate  $\rho^2 = \cos 2\theta$
- b) Find the bilinear transformation which sends the points  $z = 0, 1, \infty$  in to the points w = -5, -1, 3 respectively. What are the invariant points of the transformation? (05)
- c) Following table gives the values of x and y:

(04)

Ī	х	1.0	1.05	1.10	1.15	1.20	1.25	1.30
	у	1.00	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

Find  $\frac{dy}{dx}$  for x = 1.05 using forward difference.

## Q-5 Attempt all questions

**(14)** 

(05)

- a) Solve by Gauss Jordan method (05)
- 5x 2y + 3z = 18, x + 7y 3z = -22, 2x y + 6z = 22.

**b)** Solve the equation 27 x + 6y - z = 85, 6x + 5y + 2z = 72, x + y + 54z = 110by Gauss – Seidel method.

c) If  $\vec{F} = (x + y + 1)i + j - (x + y)k$  find  $\vec{F} \cdot curl \vec{F}$ . (04)

## Q-6 Attempt all questions

(14)

- Verify Green's theorem for the function  $\vec{F} = (x + y) i + 2xy j$  and C is the rectangle in XY -plane bounded by x = 0, y = 0, x = a, y = b. (07)
- Verify Stokes's theorem for  $\vec{A} = (2x y)i yz^2j y^2zk$ , where S is the upper half surface of sphere  $x^2 + y^2 + z^2 = 1$  and C is its boundary. (07)

## Q-7 Attempt all questions

(14)

(05)

- Use the fourth order RungeKutta method to solve  $\frac{dy}{dx} = y \frac{2x}{y}$ , y(0) = 1.

  Evaluate the value of y when x = 0.1
- **b)** Find the value of y for x = 0.1 by Picard's method, given that



$$\frac{dy}{dx} = \frac{y - x}{y + x}, y(0) = 1$$

c) Following table gives the values of x and y:

x	30	35	40	45	50
у	15.9	14.9	14.1	13.3	12.5

Find value of x corresponding to y = 13.6

## Q-8 Attempt all questions

(14) (05)

(05)

(04)

a) Construct Newton's forward interpolation polynomial for the following data:

X	4	6	8	10
Y	1	3	8	16

Use it to find the value of y for x = 5.

**b)** Use Lagrange's interpolation formula to find the value of y when x = 10, if the values of x and y are given below:

X	5	6	9	11
у	12	13	14	16

Divide the range into 10 equal parts, find the approximate value of  $\int_0^{\pi} \sin x \ dx$  by Simpson's  $\frac{1}{3}$  rule. (04)